# TEMPERATURE OF A THIN BODY WITH PERIODIC VARIATION OF THE HEAT TRANSFER COEFFICIENT AND THE TEMPERATURE OF THE SURROUNDING MEDIUM

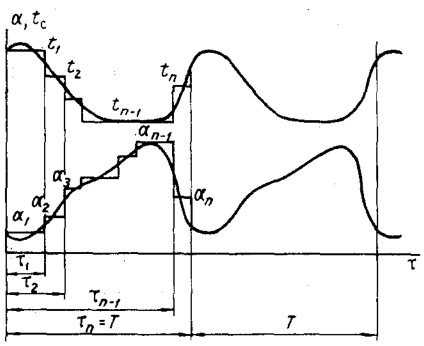
Ya. A. Levin and I. M. Prikhod'ko

Inzhenerno-Fizicheskii Zhurnal, Vol. 10, No. 2, pp. 225–227, 1966

UDC 536.244

A solution is given to the problem of heating of a thermally thin body with arbitrary periodic variation of the heat transfer coefficient and of the temperature of the surrounding medium.

In modern technology the problem arises of heating bodies which are thin, in the thermal sense, with arbitrary periodic laws of variation of heat transfer coefficient and of temperature of the surrounding medium. A mathematically similar problem may be formulated thus.




Variation with time of the heat transfer coefficient and the temperature of the surrounding medium.

A thin body with initial temperature $t_0$ is located in a homogeneous medium whose heat transfer coefficient and temperature vary with time according to arbitrary periodic laws. To determine the body temperature as a function of time, we require to find the integral of the differential equation

$$dt = k\alpha(\tau)\,[t_c(\tau) - t(\tau)]\,d\tau, \qquad (1)$$

where

$$k = F\psi/\gamma cV.$$

The coefficient $\psi$ appearing in the parameter k characterizes the nonuniformity of temperature distribution in the body. This problem may be of interest in designing regenerators filled with thin metal ribbon or wire, and in other applications [2].

The solution of the problem is accomplished in two stages. In the first stage we seek a value of the temperature of the thin body during the first period T of oscillation of heat transfer coefficient and medium temperature. For this purpose functions $\alpha(\tau)$ and $t_c(\tau)$ are approximated by stepwise straight lines, as shown in the figure.

Since within each section of the approximating curve the temperature of the medium and the heat transfer coefficient are constant, while the initial body temperature in each successive section is equal to that of the body at the end of the preceding one, we may write an expression for the body temperature at the end of any section of the approximating curve

$$t(\tau_i) = t_i - [t_i - t(\tau_{i-1})]\exp(-k\alpha_i\tau_i). \qquad (2)$$

Following transformation of (2) at i = n, we obtain the temperature of the body at the end of the first period:

$$t(\tau_n) = t_1[1 - \exp(-k\alpha_1\tau_1)] \times$$
$$\times \exp\{-k[\alpha_2(\tau_2-\tau_1) + \alpha_3(\tau_3-\tau_2) + \ldots$$
$$\ldots + \alpha_n(\tau_n - \tau_{n-1})]\} + t_2\{1 - \exp[-k\alpha_2(\tau_2-\tau_1)]\} \times$$
$$\times \exp\{-k[\alpha_3(\tau_3-\tau_2) + \alpha_4(\tau_4 -$$
$$-\tau_3) + \ldots + \alpha_n(\tau_n - \tau_{n-1})]\} + \ldots$$
$$\ldots + t_n\{1 - \exp[-k\alpha_n(\tau_n - \tau_{n-1})]\} +$$
$$+ t_0\exp\{-k[\alpha_1\tau_1 + \alpha_2(\tau_2-\tau_1) + \ldots + \alpha_n(\tau_n - \tau_{n-1})]\},$$

or

$$t(T) = a + t_0 b,$$

where b < 1, since k, $\alpha_1$, $\alpha_2$, ..., $\alpha_n$, $\tau_1$, $(\tau_2 - \tau_1)$, ..., $(\tau_n - \tau_{n-1})$ are positive quantities.

Thus, the body temperature at the end of the first period with the $t_1$, $t_2$, ..., $t_n$, $\alpha_1$, $\alpha_2$, ..., $\alpha_n$ known in the sections is determined by its initial temperature $t_0$. This same thesis may also be extended to any other, m-th, period of oscillation $\alpha(\tau)$ and $t_c(\tau)$:

$$t(T)_m = a + t(T)_{m-1}\,b. \qquad (3)$$

In the second stage of the solution there is the problem of determining the body temperature at the end of any period of oscillation of the heat transfer coefficient and of the temperature of the surrounding medium, i.e., essentially at any time. For this we use the properties of second-order recursion relations [1]. From analogy with (3), the body temperature at the end of the (m+1)-th and (m+2)-th periods will be determined, respectively, by

$$t(T)_{m+1} = a + t(T)_m b,$$
$$t(T)_{m+2} = a + t(T)_{m+1}\,b,$$

whence

$$t(T)_{m+2} - (1+b)\,t(T)_{m+1} + b\,t(T)_m = 0. \qquad (4)$$

Since (4) is the reciprocal equation of a second-order recursion relation, the corresponding characteristic equation

$$z^2 - (1 + b)z + b = 0 \qquad (5)$$

will allow us to find the common term of the series examined in the form

$$t(T)_m = C_1 z_1^m + C_2 z_2^m,$$

where $z_1 = 1$, $z_2 = b$ are the roots of the characteristic equation (5), and $C_1$ and $C_2$ are constants determined from the initial conditions; we may then obtain an expression for the body temperature at the end of any period

$$t(T)_m = t_0 b^m + [a/(1 + b)](1 - b^m). \qquad (6)$$

Here

$$b = \exp\{-k[\alpha_1\tau_1 + \alpha_2(\tau_2 - \tau_1) + \ldots + \alpha_n(\tau_n - \tau_{n-1})]\},$$

$$a = t_1[1 - \exp(-k\alpha_1, \tau_1)]\times$$

$$\times \exp\{-k[\alpha_2(\tau_2 - \tau_1) + \alpha_3(\tau_3 - \tau_2) + \ldots$$

$$\ldots + \alpha_n(\tau_n - \tau_{n-1})]\} + t_2\{1 - \exp[-k\alpha_2(\tau_2 - \tau_1)]\} \times$$

$$\times \exp\{-k[\alpha_3(\tau_3 - \tau_2) + \alpha_4(\tau_4 -$$

$$- \tau_3) + \ldots + \alpha_n(\tau_n - \tau_{n-1})]\} + \ldots$$

$$\ldots + t_n\{1 - \exp[-k\alpha_n(\tau_n - \tau_{n-1})]\}.$$

Since $b < 1$, in quasi-steady conditions ($m \to \infty$) the body temperature is

$$t(T)_\infty = a/(1 + b). \qquad (7)$$

The solution obtained is simple to use and accurate for thin bodies, if $\alpha(\tau)$ and $t_c(\tau)$ are stepwise periodic functions. For other periodic laws of variation of $\alpha(\tau)$ and $t_c(\tau)$, the accuracy of the solution depends on how well the curves are approximated by stepwise straight lines.

NOTATION

$t(\tau)$—body temperature; $\alpha(\tau)$, $t_c(\tau)$—heat transfer coefficient and temperature of surrounding medium, respectively; $\tau$—time; T—period of oscillation of heat transfer coefficient and medium temperature; F, V—surface area and volume of body, respectively; $\gamma$, c—specific weight and thermal conductivity of material of body, respectively.

REFERENCES

1. A. I. Markushevich, Recursion Relations [in Russian], GITTL, 1950.

2. E. P. Plotkin and V. I. Molchanov, IFZh, no. 2, 1963.

23 May 1965